Chapter 12

12.1

$$T' = \lambda_R Q \left(1 - e^{-t/\tau_R}\right)$$

50m ocean $\Rightarrow$ $C = 50m \cdot 4.2 \times 10^{6}\ J K^{-1} m^{-2} m^{-1}$

$= 2.1 \times 10^{8}\ J K^{-1} m^{-2}$

$\tau_R = C \lambda_R = 2.1 \times 10^{8}\ J K^{-1} m^{-2} \cdot 1.0\ °K/(Wm^{-2})$

$= 2.1 \times 10^{8}\ s = 6.65$ years

$$T' = 1.0\ °K\,Wm^{-2} \times 4 Wm^{-2} \left(1 - e^{-2/6.65}\right)$$

$= 4°K \cdot (0.26)$

$= \sim 1°K$ 25% in 2 years

$T'(t=\infty) = 4°K$

12.3

299 − 301 = 2°C at surface

at 300mb ~ 4°C or so

Chapter 12
Exercise 5

12.5

(12.8) $T' = \lambda_R Q_t \{ t + \tau_R (e^{-t/\tau_R} - 1) \}$

assume t is big enough that this term can be neglected

100m

λ_R either 0.5 or 1.0 K/Wm^{-2}

$T'_{1.0} - T'_{0.5} = 0.5°K$

$\lambda_{R_1} Q_t (t - \tau_{R_{1.0}}) - \lambda_{R_{0.5}} Q_t (t - \tau_{R_{0.5}}) = 0.5$

$Q_t \{ \lambda_{R_1} (t - \tau_{R_{1.0}}) - \lambda_{R_{0.5}} (t - \tau_{R_{0.5}}) = 0.5 / Q_t$

$\lambda_{R_1} (t - \tau_{R_{1.0}}) - \lambda_{R_{0.5}} (t - \tau_{R_{0.5}}) = 0.5 / Q_t$

$\lambda_{R_1} t - \lambda_{R_{0.5}} t - \lambda_{R_1} \tau_{R_{1.0}} + \lambda_{R_{0.5}} \tau_{R_{0.5}} = \frac{0.5}{Q_t}$

$t (\lambda_{R_1} - \lambda_{R_{0.5}}) - \lambda_{R_1} \tau_{R_{1.0}} + \lambda_{R_{0.5}} \tau_{R_{0.5}} = \frac{0.5}{Q_t}$

$t = \left(\frac{0.5}{Q_t} + \lambda_{R_1} \tau_{R_{1.0}} - \lambda_{R_{0.5}} \tau_{R_{0.5}} \right) / (\lambda_{R_1} - \lambda_{R_{0.5}})$

$$Q_t = \frac{4\ \mathrm{Wm^{-2}}}{50\ \mathrm{year}} = \frac{4}{50}\ \mathrm{Wm^{-2}yr^{-1}}$$

$$= ~~0.8 \times 10~~\ \mathrm{Wm^{-2}yr^{-1}}$$

$$0.08\ \mathrm{Wm^{-2}yr^{-1}}$$

$$\tau_{R_{1.0}} = C\lambda_{R_{1.0}} \qquad \lambda_R ? = C\lambda_R^2$$

$$C = 100\ \mathrm{m\ of\ Ocean}$$
$$= 4.2\times10^{8}\ \mathrm{JK^{-1}m^{-2}}$$

$$\tau_{R_{1.0}} = \frac{4.2\times10^{8}\ \mathrm{JK^{-1}m^{-2}}}{86400\times365.25\ \mathrm{s\,yr^{-1}}} \cdot 1.0\ ^\circ\mathrm{K\,Wm^{-2}}$$

$$= 13.3\ \mathrm{years}$$

$$\tau_{R_{0.5}} = 6.65\ \mathrm{years}$$

$$t = \left(\frac{0.5\ ^\circ\mathrm{K}}{0.08\ \mathrm{Wm^{-2}yr^{-1}}} + 1.0\ ^\circ\mathrm{K}(\mathrm{Wm^{-2}})^{-1}\cdot 13.3\ \mathrm{yrs} - 0.5\ ^\circ\mathrm{K}(\mathrm{Wm^{-2}})^{-1}\cdot 6.65\ \mathrm{yrs}\right) \Big/ (1.0-0.5)\ ^\circ\mathrm{K}(\mathrm{Wm^{-2}})$$

$$= \left(6.25\ \mathrm{yrs} + 13.3\ \mathrm{yrs} - 3.325\ \mathrm{yrs}\right) / 0.5$$

$$= 32.45\ \mathrm{years} \qquad e^{32.45/13.3} \sim e^{-2}\ \text{neglect! OK}$$

HINTS ON HW 12.4

a) Find the emissivity

First you need the Ozone irradiance absorption

Go back to chapter 3 and get (3.38)

$$\frac{\partial T}{\partial t} = -\frac{1}{\rho C_p}\frac{dF}{dz}$$

Solve for dF in $W m^{-2}$

Next take this heating in $W m^{-2}$ and put in (12.1) and solve for ε

b) Use (12.2)

12.4 Solution

a) First need solar absorption in $W m^{-2}$
use 3.38

$$\frac{\partial T}{\partial t} = -\frac{1}{c_p \rho}\frac{dF}{dz}$$

$$dF = c_p \rho \, dz \, \frac{dT}{dt}$$

$$= 1004 \, J \, kg^{-1} \, {}^\circ K^{-1} \cdot 4\times10^{-4} \, kg \, m^{-3} \cdot 10^{3} \, m \cdot \frac{10 \, {}^\circ K \, day^{-1}}{86400 \, s \, day^{-1}}$$

$$= 0.046 \, W m^{-2}$$

Next use 12.1 to solve for emissivity

$$\varepsilon = \frac{SO_3}{2\sigma T_{STRAT}^4 - OLR} \qquad OLR = \sigma T_e^4$$

$$= \frac{0.046 \, W m^{-2}}{2\sigma 280^4 - 240 \, W m^{-2}}$$

$$\varepsilon = 1\times10^{-4}$$

b) Use 12.2 to Solve for T_{STRAT}

$$T_{STRAT} = \sqrt[4]{\frac{(1.12\times10^{-4})^{-1} SO_3 + OLR}{2\sigma}}$$

$$= \sqrt[4]{\frac{0.046 \, W m^{-2} \, (1.12\times10^{-4})^{-1} + 240}{2\sigma}} = \sqrt[4]{\frac{410.7 + 240}{2\sigma}} = 275.2 \, K$$

Problem 12.4b Another Way to solve

$$(12.1) \quad S_{O_3} + \varepsilon\sigma T_e^4 = 2\varepsilon\sigma T_{ST}^4$$

$$\frac{S_{O_3}}{\varepsilon} = 2\sigma T_{ST}^4 - OLR$$

$$= 2\sigma\, 280^4 - 240$$

$$= 697 - 240 = 457\ \mathrm{W\,m^{-2}}$$

$$T_{ST} = \sqrt[4]{\frac{457 + 240}{2\sigma}} = 280\mathrm{K} \text{ as observed}$$

Now Increase ε to $1.12\,\varepsilon$

$$T_{ST} = \sqrt[4]{\frac{\frac{457}{1.12} + 240}{2\sigma}} = \underline{\underline{275\,K}}$$

In this way you avoid the intermediate calculation of ε.