Chapter 3, # 1, 3

Chapter 3

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77 Slides

#1 $M = 1\,g\,kg^{-1}$
$k = 5\,m^2\,kg^{-1}$

$$\tau = \frac{p_s}{g} M_a k_{abs}\, e^{-z/H}$$

$$H = \frac{RT}{g} = \frac{287 \cdot 260}{9.81} = 7606\,m$$

$$\tau = \frac{1.013 \times 10^5\,Pa}{9.81\,m s^{-2}}\, 0.001 \frac{kg}{kg}\, 5\,m^2 kg^{-1}\, e^{-z/H}$$

$$1 = 51.63\, e^{-z/H}$$

$$e^{-z/H} = \frac{1}{51.63}$$

$$-z/H = \ln\left(\frac{1}{51.63}\right)$$

$$z = -H \ln\left(\frac{1}{51.63}\right) = 29{,}999 \sim 30\,km$$

$$\rho = \rho_0 e^{-z/H}$$
$$\rho_0 = \frac{p}{RT} = \frac{1.013 \times 10^5\,Pa}{287 \cdot 260} = 1.36\,kg\,m^{-3}$$

#3 $$\frac{\partial F}{\partial z} = \frac{\tau}{H} \frac{F_\infty}{\mu} e^{-\tau/\mu}$$

$$= \frac{1}{H} \frac{1}{20} \frac{1367}{4} e^{-1} = \frac{1}{7606} \frac{1}{20} \frac{1367}{4} e^{-1} = 8.27 \times 10^{-4}\,W m^{-3}$$

=

$$\frac{\partial T}{\partial t} = \frac{1}{\rho c_p} \frac{\partial F}{\partial z} = \frac{1}{\rho_0 e^{-30/7.606}\, c_p} 8.27 \times 10^{-4}\,W m^{-3} = \frac{8.27 \times 10^{-4}}{0.0263\,kg\,m^{-3}\; 1004\,J\,K^{-1} kg^{-1}}$$

$$= 0.313 \times 10^{-4}\,K s^{-1}$$

$$86400 \times 0.313 \times 10^{-4} = \underline{2.7\ K\,day^{-1}}$$

Chapter 3 : 3.2 Problem 2, Chapter 3

#2
3.2

$$\left.\frac{dT}{dt}\right|_{Rad} = \frac{k_{abs} M_a}{c_p \mu} F$$

$$F = F_\infty e^{-1/\mu} \times 0.05 = \frac{1367}{4} W m^{-2} e^{-1} \, 0.05$$

$$= \frac{25}{4} W m^{-2} =$$

$$\left.\frac{dT}{dt}\right|_{Rad} = \frac{5 m^2 kg_a^{-1} \, 10^{-3} kg_a/kg \, \frac{25}{4} W m^{-2}}{1004 J kg^{-1} {}^\circ K^{-1} \cdot 1} = \frac{1.25 \times 10^{-4}}{4} {}^\circ K s^{-1}$$

$$= \underline{2.7 {}^\circ K \, day^{-1}}$$

One scale height above:

$$\tau = \frac{p_s}{g} M_a K_{abs} e^{-z/H}$$

$$= \frac{1.025 Nm^{-2} 10^5}{9.81 m s^{-2}} \cdot 10^{-3} \frac{kg_a}{kg} \, 5 \, m^2/kg_a \, e^{-z/H}$$

$$= 52.24 \, e^{-z/H}$$

$\tau = 1$ at $\quad 1 = 52.24 \, e^{-z/H}$

$$\ln\left(\frac{1}{52.24}\right) = -z/H$$

$$z = -H \ln\left(1/52.24\right)$$

$$= \underline{30{,}088 \, m}$$

→

3.2 Cont'd.

One Scale Height above

$$z_{+1} = 30{,}088 + 7606 = 37{,}695\,m$$

$$z_{-1} = 30{,}088 - 7606 = 22{,}482$$

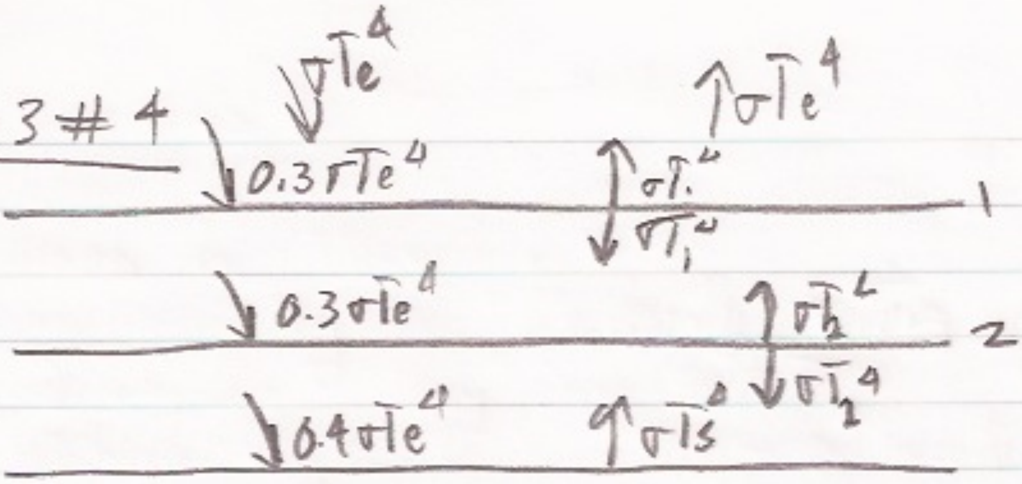
$$\left.\frac{dT}{dt}\right)_{Rad} = \frac{5\times10^{-3} \cdot 17.09\,W\,m^{-2}}{1004} e^{-\tau}$$

$$\tau(z_{+1}) = 5224\, e^{-\frac{37695}{7606}} = 0.3678$$

$$\tau(z_{-1}) = 5224\, e^{-22482/7606} = 2.718$$

$$\left.\frac{dT}{dt}\right)_{Rad} = 8.51\times10^{-5} e^{-\tau} = \begin{cases} \underline{5.1\ {}^\circ K\ day^{-1}} & z_{+1} \\ \underline{0.48\ {}^\circ K\ day^{-1}} & z_{-1} \end{cases}$$

Chap 3 # 4



in = out

TOA $\sigma T_e^4 = \sigma T_1^4 = T_1 = T_e$

Level 1 $0.3\sigma T_e^4 + \sigma T_2^4 = 2\sigma T_1^4$

$\Rightarrow T_2^4 = 1.7\,\sigma T_e^4$ $\quad T_2 = \sqrt[4]{1.7}\,T_e$

Level 2 $0.3\sigma T_e^4 + \sigma T_1^4 + \sigma T_s^4 = 2\sigma T_2^4$

$\Rightarrow T_s = \sqrt[4]{3.4-1.3}\quad T_s = \sqrt[4]{2.1}\,T_e$

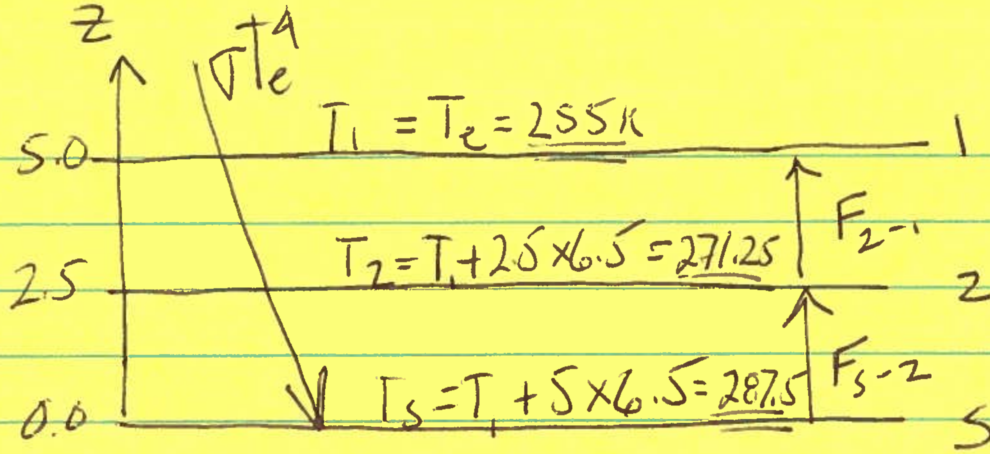
Surface $0.4\sigma T_e^4 + \sigma T_2^4 = \sigma T_s^4$

$\Rightarrow T_s = \sqrt[4]{2.1}\,T_e$

	Case 1	Case 2
T_1	255	255 K
T_2	303	291.2 K
T_s	335	307 K

colder Surface
Much more stable
less lapse rate

Problem 3.5



IN = out

TOA $\quad \sigma T_e^4 = \sigma T_1^4$

L-1 $\quad \sigma T_2^4 + F_{2-1} = 2\sigma T_1^4$

L-2 $\quad \sigma T_s^4 + \sigma T_1^4 + F_{s-2} = 2\sigma T_2^4 + F_{2-1}$

Surface $\quad \sigma T_e^4 + \sigma T_2^4 = \sigma T_s^4 + F_{s-2}$

L-1 $\quad F_{2-1} = 2\sigma T_1^4 - \sigma T_2^4 = \underline{\underline{172.5\ Wm^{-2}}}$

surf $\quad F_{s-2} = \sigma T_e^4 + \sigma T_2^4 - \sigma T_s^4 = \underline{\underline{159.3\ Wm^{-2}}}$

How do fluxes Compare:

$LE + SH = 0.29 \times 342\ Wm^{-2} = \underline{99\ Wm^{-2}}$

We got $159.3\ Wm^{-2}$ Way too big by 50%

Net longwave $= 0.21 \times 342\ Wm^{-2} = \underline{72\ Wm^{-2}}$

We got $\sigma T_s^4 - \sigma T_2^4 = \underline{\underline{80\ Wm^{-2}}}$

Pretty Close!

Chapter 3, #4

3.6

Pure Radiative Equil
Same as given on page 63

255 303 335 K
T_1 T_2 T_S

Suppose you move top up, then T_S and T_2 increase, while $T_1 = T_e$ is unchanged

$$C_S = \sigma(T_e^4 + T_2^4 - T_S^4)$$

if $T_e = \text{const}$ $T_S > T_2$ then
C_S decreases as the top is moved up

$$\Delta C_S = \left(\sigma T_e^4 - \sigma T_e^4 + T_2^4 + (T_2^4 + 4\sigma T_2^4 \Delta T) - \left(T_S^4 + (T_S^4 + 3\sigma T_S^4 \Delta T\right)\right)$$

$$\cong 4\sigma\left(\left(T_2^3 - T_S^3\right)\Delta T\right.$$

$$\cong 4\sigma(\quad)\Delta T$$

$$= -0.86\ Wm^{-2}K^{-1}\ \Delta T$$

It makes sense for the required surface non-radiative fluxes to decrease as you move the top up and lower trop & surface temp increase.

Chapter 3, #7

3.7.

$$\Delta\alpha_p = 0.6 - 0.1 = 0.5$$

$$S = 400\ \mathrm{W\,m^{-2}}$$

$$OLR_{clear} = 280\ \mathrm{W\,m^{-2}}$$

use 3.64

$$T_{ZCT} = \left(\frac{-400 \cdot 0.5 + 280}{\sigma}\right)^{1/4}$$

$$= \left(\frac{80}{\sigma}\right)^{1/4} = \underline{194\,K}$$ Tropical Tropopause Temps

$$z_{CT} = \frac{300K - 194K}{5} = \underline{21.2\,km}$$ a bit high

$$z_{CT} = \frac{300 - 194}{5.5} = \underline{19.3\,km}$$