Chapter 5 HW, #1

p 134 #1

precip over land 75 cm/year

a) $8.062 \times 10^{6}\ km^{3} = 8.062 \times 10^{15}\ m^{3}$

$Area = 4\pi r^{2}\ 0.3 = 1.53 \times 10^{14}\ m^{2}$

depth of soil moisture on land $\frac{8.062 \times 10^{15}\ m^{3}}{1.53 \times 10^{14}\ m^{2}} = 52.7\ m$ depth

$\frac{52.7\,m}{.75\ m/year} = \underline{\underline{70\ years}}$

b) 10% of runoff $\frac{\Delta F}{P} = 0.36$

$\frac{52.7\,m}{0.36 \times 0.1 \times 0.75\ m/year} = \underline{\underline{1952\ years}}$

$\frac{52.7}{0.1 \times 0.27} = \boxed{1952\ years}$

Chapter 5 #2

Homework: for Last Week # 2

$$E = \rho C_{DE} V (q_s^* - q_a)$$

$$\boxed{C_{DE} = 10^{-3} \quad U = 5\,m\,s^{-1} \quad \Delta T = 2}$$

$$= \rho C_{D_E} V \left(q_s^* - RH \left(q_s^* \left(1 - \left(\frac{L}{R_v T} \right) \frac{T_s - T_a}{T_s} \right) \right) \right)$$

(4.32)

PLUG & CHUG

a.) $T_s = 0°C \quad q^* = 3.75\ g\,kg^{-1} \quad RH = 50\%$

$E = 1.2\ mm\ day^{-1}$

b.) $T_s = 0°C \quad q^* = 3.75\ g\,kg^{-1} \quad RH = 100\%$

$E = 0.305\ mm\,day^{-1}$

c) $T_s = 30°C \quad q^* = 27 \quad RH = 50\%$

$E = 7.8\ mm\ day^{-1}$

d) $T_s = 30°C \quad q^* = 27 \quad RH = 100\%$

$E = 1.67\ mm\ day^{-1}$

Chapter 5 Page 134 #1 & 2

#1 $8.062\times10^{6}\ km^{3}$ = Volume of groundwater

$$\text{Area of land} = 0.3 * 4\pi r_e^{2}$$
$$= 0.3\times 4\pi\,(6370\ km)^{2}$$
$$= 1.53\times10^{8}\ km^{2}$$

$$\text{Depth} = \frac{8.062\times10^{6}\ km^{3}}{1.53\times10^{8}\ km^{2}} = 0.0527\ km \approx 52.7\ m$$

$$\text{Time} = \frac{\text{depth}}{\frac{dh}{dt}} = \frac{52.7\ m}{0.75\ m\ yr^{-1}} = \boxed{70.2\ \text{years}}$$

#2 (4.32) $LE = \rho L C_D U\left(q_s^{*}(1-RH) + RH\,B_e^{-1}\frac{c_p}{L}(T_s-T_a)\right)$

$C_D = 10^{-3}$ $U = 5\,m\,s^{-1}$ $\rho = 1.2\ kg\,m^{-3}$ ~~RH = 50~~

a) $T_s = 0\ C$ $q_s^{*} = 3.75\ g\,kg^{-1}$ $RH = 50\%$

from Fig 4.1 $B_e = 1.5$

$$LE = \rho L C_D U\left(q_s^{*}(1-RH) + RH\,B_e^{-1}\frac{c_p}{L}(T_s-T_a)\right)$$
$$= 1.2\ kg_A m^{-3}\ \cancel{2.5\times10^{6}\ J\,kg_{H_2O}^{-1}}\ 10^{-3}\ 5\,m\,s^{-1}\left(3.75\tfrac{g}{kg}(0.5) + 0.5\ \tfrac{2}{3}\ \frac{1004\ J\,kg^{-1}K^{-1}}{2.5\times10^{6}\ J\,kg_{H_2O}^{-1}}\ 2\,K\right)$$
$$= 1.2\ \cancel{2.5\times10^{6}}\ 10^{-3}\ 5\left(3.75\times0.5\times10^{-3} + 0.5\ \tfrac{2}{3}\ \frac{1004}{2.5\times10^{6}}\cdot 2\right)\ kg/m^{2}$$
$$= 1.2\ 10^{-3}\cdot 5\left(3.75\times10^{-3}\ 0.5 + 0.5\ \tfrac{2}{3}\ \frac{1004}{2.5\times10^{6}}\cdot 2\right)\ kg_{H_2O}\,s^{-1}$$
$$6\times10^{-3}\left(1.875\times10^{-3} + 2.6\times10^{-4}\right)$$
$$= 1.281\times10^{-5}\ kg_{H_2O}\,s^{-1}\,m^{-2}$$
$$= 1.281\times10^{-5}\times 86400\ s/day = \frac{1.1\ kg\ day^{-1}\ m^{-2}}{10^{3}\ kg\ m^{-3}} = 1.1\ mm\ day^{-1}$$

chapter 5, #2 (cont.) – #3

#2 contd

b) Same as a) but

$$E = \rho C_D U \left(q_s^*(0) + B_e^{-1} \frac{c_p}{L}(T_s - T_0) \right)$$

$$6\times10^{-3} \left(0 + \frac{2}{3} \frac{1004}{2.5\times10^{6}} (2) \right)$$

5.2×10^{-4}

$= 3.12\times10^{-6}\ kg\ s^{-1}\ m^{-2}$

$= 3.12\times10^{\cancel{-6}\,-9} \cdot \frac{86400}{\cancel{10^{-3}}} = \boxed{0.27\ mm\ day^{-1}}$

c) $T_s = 30°C \Rightarrow B_e = 0.3 \Rightarrow \boxed{7.8\ mm\ day^{-1}}$

d) $\boxed{1.67\ mm\ day^{-1}}$

#3 $\dfrac{SH = c_p \cancel{\rho} \cancel{C_D} \cancel{U} (T_s - T_a)}{LE = L \cancel{\rho} \cancel{C_D} \cancel{U} \left(q_s^*(1-RH) + RH\, B_e^{-1} \frac{c_p}{L}(T_s - T_a) \right)}$

$$B_o = \frac{c_p (T_s - T_a)}{L \left(q_s^*(1-RH) + RH\, B_e^{-1} \frac{c_p}{L}(T_s - T_a) \right)}$$

$$B_o = \frac{c_p}{L} \frac{T_s - T_a}{\left(q_s^*(1-RH) + RH\, B_e^{-1} \frac{c_p}{L}(T_s - T_a) \right)}$$

a) $0°C$ $B_e = 1.5$ $q_s^* \sim 3.75\ g/kg$

$\frac{c_p}{L} = \frac{1004}{2.5\times10^{6}} = 0.4\times10^{-3}$

$$B_o = \frac{c_p}{L} \left(\frac{2^\circ}{3.75\times10^{-3}\, 0.3 + 0.7\, \frac{2}{3} \frac{c_p}{L} 2^\circ} \right)$$

$$= 0.4\times\cancel{10^{-3}} \left(\frac{2}{1.125\times\cancel{10^{-3}} + 3.7\times10^{\cancel{-4}}\,{}^{-1}} \right) = 0.4 \left(\frac{2}{1.125 + .37} \right) = \boxed{0.535}$$

p. 135 #3

Chapter 5, #3

$$\frac{SH}{LE} = \frac{\rho c_p \epsilon_D U (T_s - T_a)}{\rho L \epsilon_D U \left(q_s^*(1-RH) + RH B_e^{-1} \frac{c_p}{L}(T_s - T_a) \right)}$$

$$= \frac{c_p}{L} \frac{\Delta T}{q_s^*(1-RH) + RH B_e^{-1} \frac{c_p}{L} \Delta T}$$

$$= \frac{\Delta T}{\frac{L}{c_p} q_s^* \underset{0.3}{(1-RH)} + RH B_e^{-1} \Delta T} = \frac{2}{7.4 \times 10^2 \cdot q_s^* + \frac{1.4}{B_e}}$$

$$= \frac{1}{374 \cdot q_s + \frac{0.7}{B_e}}$$

T	q_s^*	B_e	Bo
0	3.75	1	0.476
15	10	0.6	0.204
30	27×10^{-3}	0.15	0.668

→

p135 #3

Chapter 5, #3 (cont.)

$$B_o = \frac{c_p}{L}\frac{T_s - T_a}{q_s^* - \cancel{B}q_a}$$

$$q_a = \overset{RH}{0.7}\, q_a^* = \overset{RH*}{0.7}\left(q_s^*\left(1-\left(\frac{L}{R_vT}\right)\frac{T_s-T_a}{T_s}\right)\right)$$

$$= \frac{c_p}{L}\frac{T_s - T_a}{q_s^* - RH\, q_s^*\left(1-\left(\frac{L}{R_vT}\right)\frac{T_s-T_A}{T_s}\right)}$$

$$B_o = \frac{c_p}{L}\left(\frac{\cancel{T_s-T_A}\,\Delta T}{q_s^*\left(1 - RH*\left(1-\left(\frac{L}{R_vT}\right)\frac{\Delta T}{T_s}\right)\right)}\right) \qquad \frac{1004}{2.5\times10^6}$$

T	q_s^*	$\left(\frac{L}{R_vT_s}\right)$	$\frac{\Delta T}{T_s}$		
0°C	3.75	$\frac{2.5\times10^6}{461\cdot273}$ (19.9)	0.00733		0.539
15°C	10.	18.8	0.00694		0.205
30°C	27	17.9	0.00660	0.1935	0.078

→

Chapter 5, #3 (cont.)

$$B_o = \frac{c_p}{L}\frac{T_S - T_A}{q_s^* - q_a}$$

$$B_o = B_e\left(1 - \frac{q_a^* - q_a}{q_s^* - q_a}\right) \qquad q_a = 0.7\, q_a^*$$

$$= B_e\left(1 - \frac{q_a^*(0.3)}{q_s^* - q_a^*(0.7)}\right)$$

Read B_e off Fig 4.10
q^* off Fig B.1.

B.2 $\cancel{q_a^*} = \dfrac{q_s^* - q_a^*}{q_s^*} = \left(\dfrac{L}{R_v T}\right)\dfrac{\Delta T}{T}$

$$q_s^* - q_a^* = q_s^*\left(\frac{L}{R_v T}\right)\frac{\Delta T}{T}$$

$$q_a^* = q_s\left(1 - \left(\frac{L}{R_v T}\right)\frac{\Delta T}{T}\right)$$

$$q_a = q_a^*(0.7)$$

Chapter 5, #3,4,5

#3 $$\frac{SH}{LE} = \frac{\rho c_p C_D U (T_s - T_a)}{\rho L C_D U \left(q_s^*(1-RH) + RH\, B_e^{-1} \frac{c_p}{L}(T_s - T_a)\right)}$$

$$= \frac{c_p}{L} \frac{\Delta T}{q_s^*(1-RH) + RH\, B_e^{-1} \frac{c_p}{L} \Delta T}$$

T	q_s^* g/kg	B_e	Bo
0	3.75	1	0.476
15	10.0	0.6	0.204
30	~~0.75~~ 27.	0.15	0.068

4) High lats excess energy → SH (see next page)

5) Think! Soil is saturated after snow melts.

Chapter 5 #4 (on bottom)

b.) 15°C $B_e \approx 0.7$ $q_s^* \approx 9\times10^{-3}\,\mathrm{kg\,kg^{-1}}$

$$B_0 = 0.4\times10^{-3}\left(\frac{2}{9\times0.3\times10^{-3} + 0.7\,(0.7)^{-1}\cdot\frac{c_p}{L}\cdot 2}\right)$$

$$B_0 = 0.4\times\cancel{10^{-3}}\left(\frac{2}{2.7\times\cancel{10^{-3}} + 0.8\times\cancel{10^{-3}}}\right) = \boxed{0.23}$$

c) 30°C $B_e \approx 0.17$ $q_s^* = 27\,\mathrm{g\,kg^{-1}}$

$$B_0 = 0.4\times\cancel{10^{-3}}\left(\frac{2}{27\times0.3\times\cancel{10^{-3}} + 0.7\left(\frac{1}{0.17}\right)0.8\times\cancel{10^{-3}}}\right)$$

$$= 0.4\left(\frac{2}{8.1 + 3.294}\right)$$

$$\boxed{B_0 = 0.07}$$

A) In problem 3 we see that near freezing the Bowen ratio is large ~0.5 for temps that are low. So heat that is added is not all used to evaporate water, some is used to heat the air.

In tropics most all energy is used to evaporate water.

Polar regions might stay wetter, since evap is inefficient.